

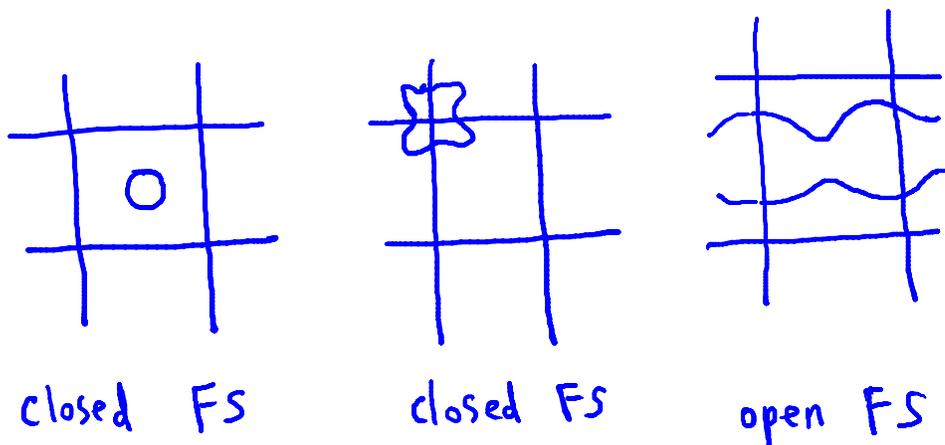
Lecture 17

Tuesday, March 01, 2011

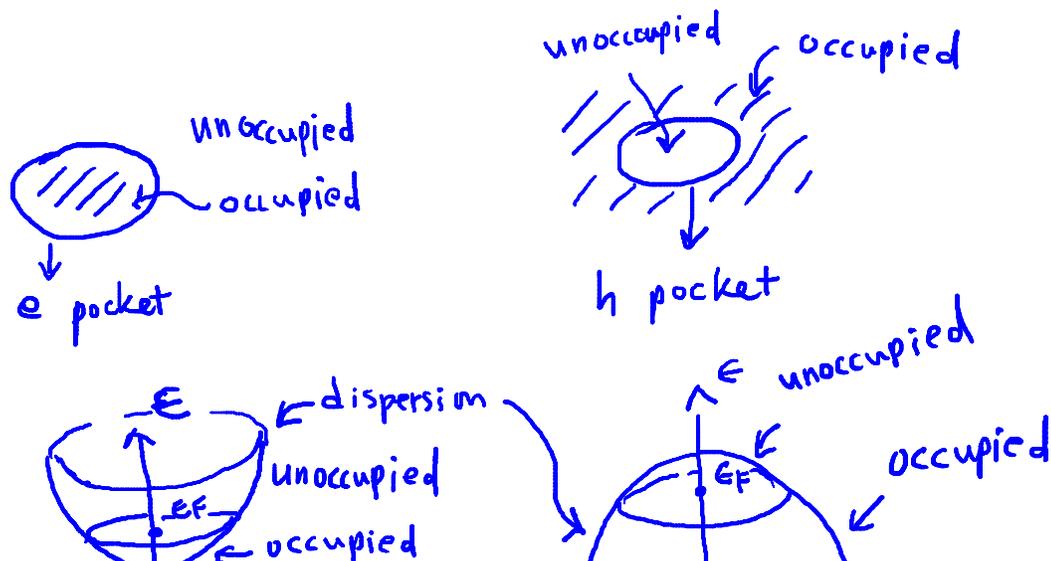
Holes and Electrons

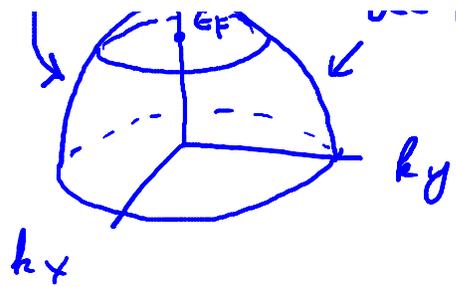
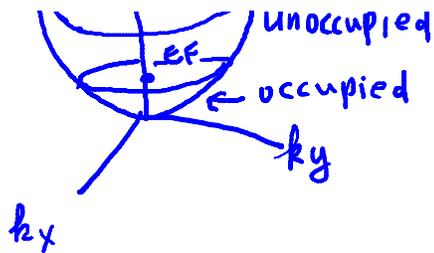
These days "topological effects" are in fashion. Topologically protected effects are robust, which is why they are important. Now, a very simple topological effect in solid state physics is the emergence of the hole as a physical entity in semi-conductors.

Let us recall how we defined the electron pocket and the hole pocket in relation to a Fermi surface. We assume that we have a closed Fermi surface, as an open Fermi surface leaves the electron or the hole nature poorly defined or dependent on the direction.



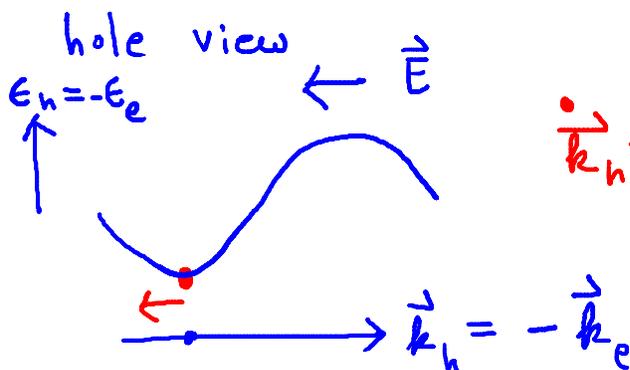
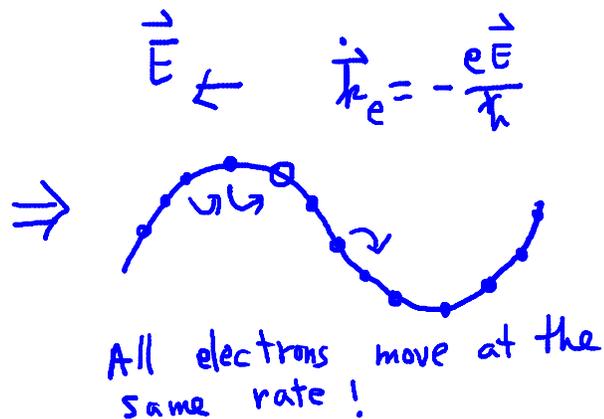
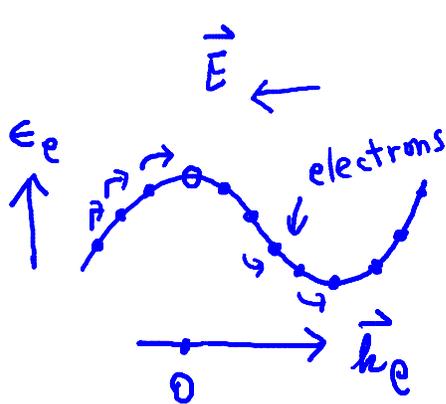
For a given closed Fermi surface, if the band dispersion that gives rise to that Fermi surface is occupied in the interior of the Fermi surface, then that is an electron pocket. If the interior is un-occupied, then that is a hole pocket.





Why is this -- e-pocket or a h-pocket -- a topological property? This is because, it is not possible to continuously deform an e-pocket to a h-pocket, and vice versa. (This topological argument is valid only in dimensions higher than one.)

By studying the following diagrams carefully, one can grasp what we really mean by "hole." The most important fact is that the concept "hole" considers the completely filled band as the reference state, or the vacuum state if you like. Thus energy of the hole = (total energy of the N - 1 state) - (total energy of the N state), where the N state corresponds to the completed filled band. The wave vector of the hole = (total wave vector of the N - 1 state) - (total wave vector of the N state). **So, to go from the electron diagram to the hole diagram, both the energy and the wave vector (momentum) need to be flipped in sign.** The same holds for the spin/orbital angular momentum. But, note that the group velocity remains the same.

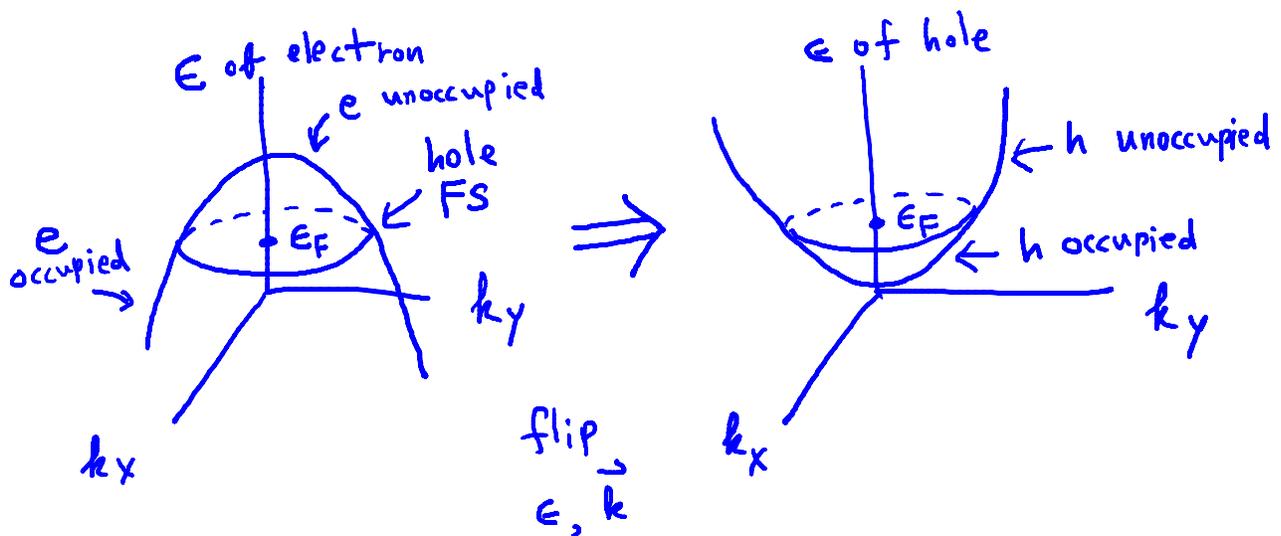


$$\vec{k}_h = \frac{e\vec{E}}{\hbar}$$

Acts like a positive charge !!

Note, by the way, that the above diagram shows that in a completely filled band, each electron will move exactly the same way in \vec{k} space. So, there is no net change. This is why materials with completely filled bands acts as though nothing happens (like no conductivity), since the overall state is not changing at all.

So it is not just the mere convenience that we call one FS an e-pocket or the other a h-pocket. They are fundamentally different. The transport phenomena arising from an electron pocket should be considered, as coming from electrons, while the transport phenomena arising from a hole pocket should be considered as holes, positively charged particles.



The hole may be defined as "the absence of an electron" but is a distinctly different particle, as measured by the probe sensitive to the charge of the particle. For instance, the Hall effect is such a measurement. Also, the relative sign of the heat conductivity and the electrical conductivity is another such measurement.